INDE 301 Engineering Economy

Chapter 7 Rate of Return Analysis: Single Alternative

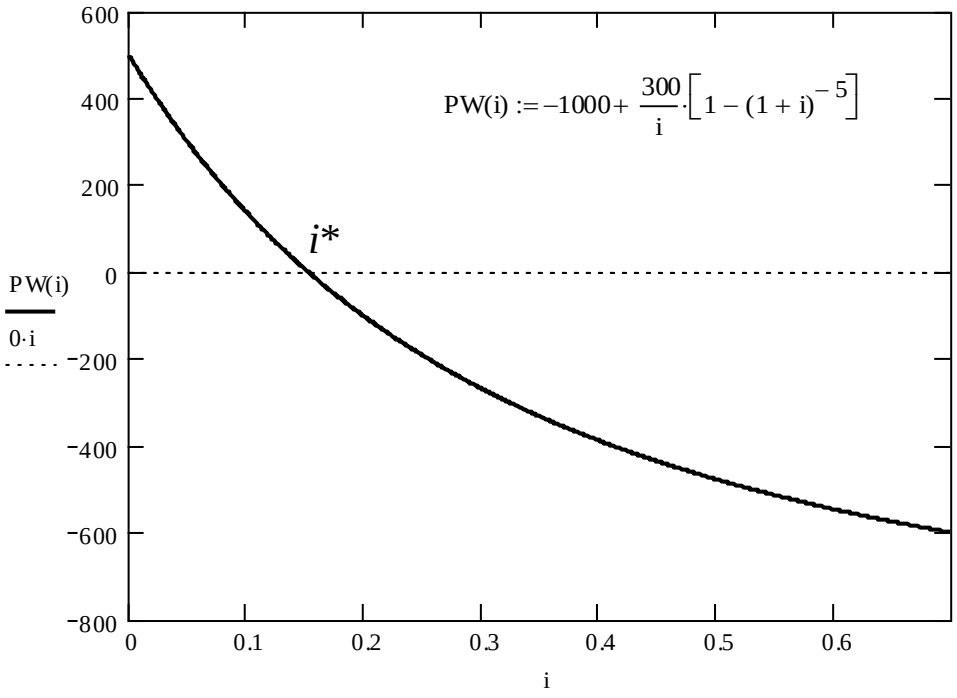
- **Introduction**
  - The rate of return (ROR) for an investment involving one single payment P and returning F after 1 year

$$\text{ROR} = i^* = F / P - 1 .$$

  - Alternatively, i^*, is given by the solution to the equation

$$-P + F / (1+i^*) = 0 \Leftrightarrow PW(i) = 0.$$

  - With more sophisticated projects involving several payments and receipts, the same equation, $PW(i) = 0$, gives the ROR.
  - E.g., for a \$1,000 investment that pays \$300 per year over 5 years, the ROR is 15.2%, as shown in the graph below.



- **Definition**
  - ROR is the rate of interest paid on the *unrecovered balance* of an investment, so that the final receipt brings the balance to exactly 0 with interest considerations.

- **Fact**
  - ROR is *not* the interest rate earned on the original investment amount.[1]

- **Deciding on an alternative based on ROR**
  - If $i^* \geq$ MARR, accept alternative.
  - If $i^* <$ MARR, reject alternative.

- **Range for *i** and relation to PW**
  - $-100\% < i^* \leq +\infty$.
  - $i^* \geq \text{MARR} \Leftrightarrow \text{PW(MARR)} \geq 0$.

- **ROR calculation using a PW or AW equation**
  - Set PW = 0 or AW = 0, this leads to
    - $-PW_D + PW_R = 0$, or
    - $-AW_D + AW_R = 0$,

    where the subscripts "*D*" and "*R*" denote disbursements (costs) and receipts.

[1] *Installment financing* is paying interest based on the loan principal (initial balance). The rate that lenders provide to promote such loans is not their true ROR. (It may put the borrower at a great disadvantage.)

- This usually involves finding the root(s) of an n^{th} degree polynomial where n is the project life.
- By hand, ROR is found by solving one of the above equations by trial and error.
- Using a computer package, such as Excel, ROR can be usually found easily.
- To solve the equation quickly, use a good starting solution.

- **Starting ROR Solution**
  - A good starting value for ROR is found as follows:
    - "Convert" all disbursements into either a single or uniform amount without considering time value of money. (This is a rough approximation.)
    - "Convert" all receipts into either a single value or a uniform series.
    - Solve the resulting PW = 0 equation which will be of the form $PW_D = PW_R \times Factor(i^*)$
  - The starting value is an approximation to ROR.
  - Another good way to get a starting solution, especially when using a computer, is to plot PW (or AW) versus i .
  - This allows visual identifying a range for values of i^*.

- **Solution by computer (Excel)**
  - In Excel, if cash flows, over *n* years, involve an initial payment, *P*, followed by a uniform series of receipts, *A*, and then a final (salvage) value, *F*.
  - Then, ROR is given by the function *RATE(n, A, P, F)*.
  - If the cash flows do not follow this particular pattern, an initial "guess" value should be determined.
  - Then, cash flows are to be inputted in detail (say in the range first_cell:last_cell).
  - Then, ROR is given by *IRR(first_cell:last_cell, guess)*.

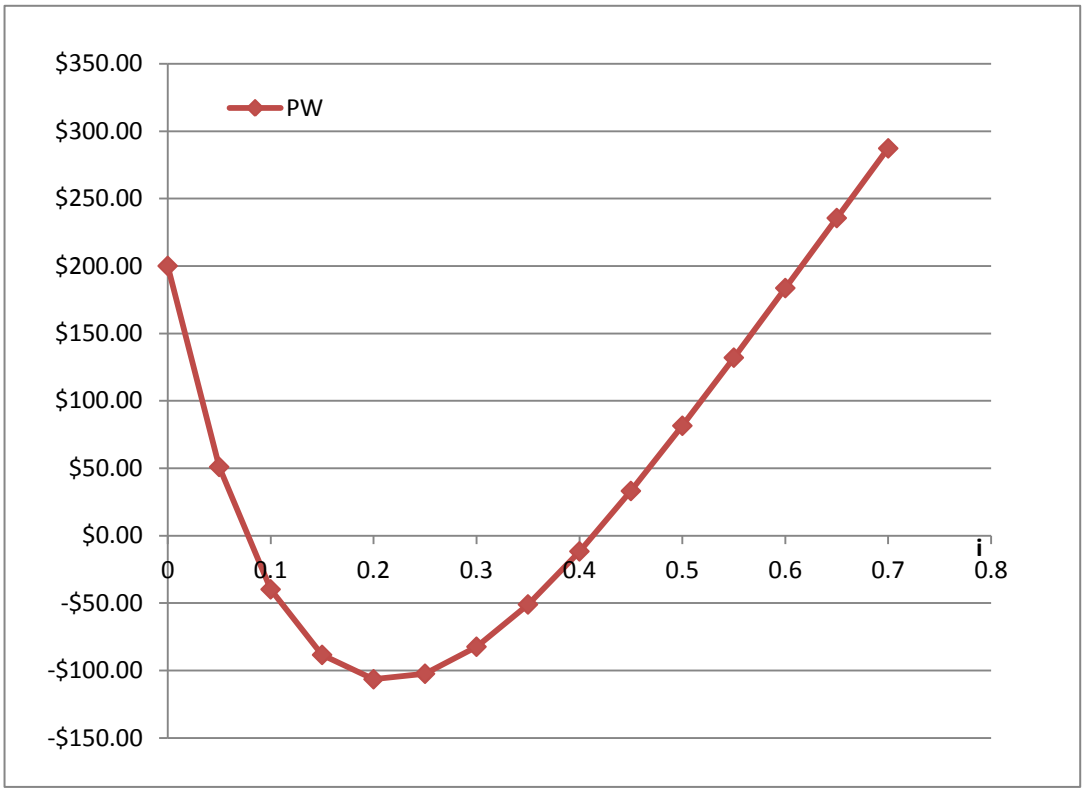
- **Advantages of using ROR**
  - No need to estimate MARR.
  - It is somewhat intuitive and easy to understand.
  - Investors like it.

- **Disadvantages of using ROR**
  - For some types of cash flows the ROR method can be computationally difficult.
  - Some cash flows will result in multiple *i** values. This raises questions as to which, if any, *i** value to use.
  - Special procedure is required when comparing multiple alternatives (Chapter 8).

- **Checking whether there are multiple ROR values**
  - There are two tests to check whether a cash flow series could have multiple ROR values
  - The tests are based on the idea that the ROR is one of the roots of the polynomial $PW(c) = \sum_{t=0}^{n} F_t c^t$, where $c = 1/(1+i)$ and F_t is the cash flow at time *t*.
  - *Descartes' rule of signs*.
    The maximum number of roots of a cash flow series is equal to the number of sign changes of cash flows.
  - *Norstrom's criterion*.
    Let $S_t = \sum_{r=0}^{t} F_r$ be the cumulative cash flows at time *t*. The cash flow series has a unique ROR if:
    - $S_0 < 0$;
    - The series $S_0, S_1, \ldots, S_n$, changes sign only once (from minus to plus).
  - When the two tests indicate that multiple ROR values *may* exist the next step is to plot PW(*i*) vs. *i* to find out how many RORs really exist.

- **Conventional / unconventional cash flow series**
  - A cash flow series is said to be conventional if it changes sign only once (from minus to plus).
  - Otherwise, the series is called unconventional.
  - Fortunately, conventional cash flows admit a unique ROR (based on Descartes' rule).
  - Unconventional cash flows may admit multiple RORs.
  - There is no universally accepted way for analyzing these cash flows.
  - Some multi-ROR analysis methods are based on assuming a *reinvestment rate* for positive cash flows.

- **Example of unconventional cash flows with multiple RORs**
  - In Example 7.4 (text), there are two ROR for the cash flow series, in $K, (2, -0.5, 8.1, 6.8).
  - The two values are 7.47% and 41.35% (found using the Excel IRR() function.)
  - Here it is not obvious which value is the right ROR.



- **Guidelines**

If the Results are	Action
Both $i^* < 0$	Discard both values
Both $i^* > 0$	Discard both values
One $i^* < 0$; One $i^* > 0$	Use $i^* > 0$ as ROR

  - Always determine the PW or AW at the MARR first for reliable measure of economic justification.
  - If PW or AW > 0 and the ROR is needed, then find the actual i^* of the project cash flows.
  - If an exact ROR is not necessary use the PW, AW, or FW methods.